Optical rotation of white light

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Plane-polarized monochromatic light is rotated in an optically active medium. The extent of the rotation is wavelength dependent, following an optical rotatory dispersion (ORD) curve. Typically, this phenomenon is studied by using a few discrete wavelengths. Here, we demonstrate optical rotation of white light. Corn syrup is used as the medium as large angles of optical rotation can be generated in compact containers. The Drude expression for ORD and Malus' law are used to predict the spectrum of the light transmitted as a function of the angle between polarizers located on either side of the sample. Despite the transmission spectrum of corn syrup in the absence of polarizers being unremarkable, optical rotation leads to a dramatic change in color because a "notch" is generated in the spectrum of the transmitted light. The extinction region can be translated across the spectrum by rotating the analyzer. The experimentally measured location of the region of maximum extinction and the color of the transmitted light are in excellent qualitative agreement with the predicted values. The experiment is ideal both as a lecture demonstration and for quantitative investigation in an undergraduate laboratory of the spectral distribution of light transmitted by a chiral medium.
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I. INTRODUCTION

Optical rotation—rotation of the plane of polarization of light as it propagates—was first discovered by Jean-Baptiste Biot in 1815, and it had an almost immediate impact by providing a quantitative measure of the purity of sugar.[1] A solution of chiral molecules (those lacking microscopic mirror symmetry) is an ideal medium for demonstrating the phenomenon. In 1848, Louis Pasteur used Biot's optical rotation technique to distinguish between left- and right-handed tartrate crystals, thereby establishing the new field of chiral chemistry.[2] The extent of the rotation achieved in optically active medium is wavelength dependent, giving rise to an optical rotatory dispersion (ORD) curve. An experimental investigation of ORD of a chiral liquid yields insight into the decomposition of linearly polarized light in a circular basis and light-matter interactions; consequently, it is a well-studied undergraduate laboratory activity, providing an attractive complement to studies of the Faraday effect.[3] An overview of the typical experimental apparatus and technique used can be found, for example, in Refs. 4 and 5 or 6. Typically, the ORD curve is assembled by using different lasers with discrete wavelengths. In this work, we present a different way to study, characterize, and quantify the ORD of a chiral solution—the optical rotation of white light or white-light polarimetry. Polarized white light is incident on the medium, with the plane of polarization of different colors being rotated by different amounts. The transmission of each color is different. Consequently, the spectrum of the transmitted light will be different from both the spectrum of the incident white light and a function of the angle of the analyzing polarizer. The colors observed in ORD are a circular birefringent analog of stress-induced linear birefringence.[7]

The medium chosen for this investigation is corn syrup, as it has a high concentration of chiral molecules, making it ideal for demonstrating and visualizing optical rotation.[8–13]

A demonstration of the principle of this experiment is seen in Fig. 1. White light is passed through a polarizer and then traverses corn syrup in a jar. The transmitted beam is incident on an array of polarizers oriented at different angles. Different wavelengths of the light are rotated by different amounts in the optically active medium; therefore, different colors are transmitted depending on the orientation of the polarizers. The wide range of colors seen in transmission as the analyzer is rotated is both surprising and dramatic, and for this reason, this experiment makes a visually attractive lecture demonstration.

The rest of this paper provides a quantitative analysis of this phenomenon and is organized as follows: Sec. II contains the relevant theoretical background, and in Sec. III, we present the apparatus, initial calibration, and how to choose the relevant experimental parameters. The results are presented and discussed in Sec. IV. Finally, conclusions are drawn in Sec. V.

II. THEORY

The physical origin of optical rotation is the difference in response of a chiral medium to opposite circularly polarized light beams—circular birefringence. We consider light of vacuum wavelength λ incident on the medium linearly polarized along the x-direction. It is easier to discuss the matter-light interaction using a basis of circular polarization. Left, $\mathcal{E}_{\rm L}$, and right, $\mathcal{E}_{\rm R}$, circular polarized waves of amplitude $\mathcal{E}_0$ propagating along z in a chiral medium can be written as[7]

$$\mathcal{E}_{\rm L} = \mathcal{E}_0 \cos(2\pi n_{\rm L} z/\lambda - \omega t)\hat{\epsilon}_x - \mathcal{E}_0 \sin(2\pi n_{\rm L} z/\lambda - \omega t)\hat{\epsilon}_y \tag{1}$$

and

$$\mathcal{E}_{\rm R} = \mathcal{E}_0 \cos(2\pi n_{\rm R} z/\lambda - \omega t)\hat{\epsilon}_x + \mathcal{E}_0 \sin(2\pi n_{\rm R} z/\lambda - \omega t)\hat{\epsilon}_y, \tag{2}$$

where $\hat{\epsilon}_x$ and $\hat{\epsilon}_x$ are the unit vectors along the x and y directions, respectively; ω is the optical angular frequency; and

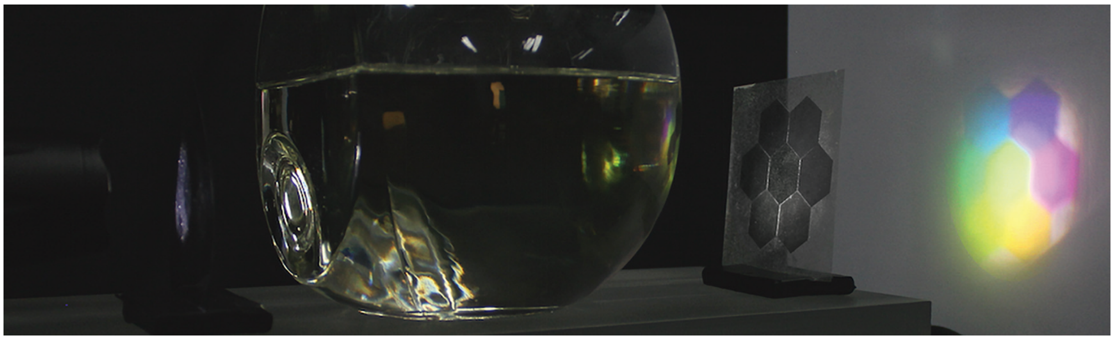

Fig. 1. Image illustrating white-light polarimetry in corn syrup. Light from a white-light source (on the left) is shone through a linear polarizer and then enters a glass jar containing corn syrup. An array of polarizers with different orientations is placed in the output and produces the colored filtering effect observed on the screen (right).

n_L and n_R are the refractive indices of the left and right circular polarization components, respectively. These components travel at different speeds in a chiral medium, $n_L \neq n_R$. We note that the field incident on the medium, polarized along x, can be written as $\mathcal{E}_{in} = (\mathcal{E}_L + \mathcal{E}_R)/2$. We also note that the analysis depends on the assumption of linearity in the light-matter interaction; the different polarization components with differing wavelength are assumed to propagate independently while simultaneously interacting with the medium. Each of the circular components of Eq. (1) and Eq. (2) accumulates a different phase in the chiral medium. If we assume that differential attenuation of the two components is negligible, then after traversing a medium of length ℓ, the light field output is

$$\mathcal{E}_{out} = \mathcal{E}_0 \left(\cos 2\pi\Delta n\,\ell/\lambda\,\hat{\epsilon}_x - \sin 2\pi\Delta n\,\ell/\lambda\,\hat{\epsilon}_y\right) \times \cos\left(2\pi n\ell/\lambda - \omega t\right). \tag{3}$$

Here, we have introduced $n = (n_L + n_R)/2$ and $\Delta n = (n_L - n_R)/2$. We recognize Eq. (3) as linearly polarized light, which has its plane of polarization rotated by an angle α after traversing the chiral medium, where

$$\alpha = \frac{\pi(n_L - n_R)\ell}{\lambda}. \tag{4}$$

It is well known[14] that the rotation angle, in addition to being proportional to the length of the medium, is proportional to the concentration, c, of the solution. Therefore, the specific rotation, $[\alpha]_\lambda^T$, is defined as

$$[\alpha]_\lambda^T = \frac{\alpha}{\ell c}. \tag{5}$$

The superscript T denotes that the specific rotation is temperature dependent, and the subscript indicates that the rotation is a function of wavelength. Typically, investigations of optical rotation in sugar solutions use visible light; thus, the light frequency is substantially smaller than the resonance frequency. In this regime, the Drude expression[15] for the wavelength dependence of the specific rotation is used,

$$[\alpha]_\lambda^T = \frac{A}{\lambda^2 - \lambda_0^2}, \tag{6}$$

where A is the rotation constant and λ_0 the dispersion constant. Plotting specific rotation as a function of wavelength yields the optical rotatory dispersion curve. For corn syrup, it has previously been established[13] that the rotation and dispersion constant—assuming a density of 1 g/cm^3—are $A = (4.89 \pm 0.02) \times 10^7\,^\circ\,\mathrm{nm}^2\ \mathrm{dm}^{-1}\,\mathrm{g}^{-1}\,\mathrm{cm}^3$ and $\lambda_0 = (166 \pm 6)$ nm, respectively.

The polarization state of the light transmitted by the chiral medium is determined by passing the output through a second polarizer—the analyzer. Malus' law predicts that the intensity, $\mathcal{I}$, of light with wavelength λ transmitted by a linear polarizer inclined at an angle $\theta' = \theta + \alpha$ with respect to the plane of polarization of incident light of intensity $\mathcal{I}_0$ is

$$\mathcal{I} = \mathcal{I}_0 \cos^2(\theta + \alpha), \tag{7}$$

where θ is the angle of the analyzer that can be adjusted in the experiment and α is a function of wavelength given by Eq. (6). For white light, the predicted intensity spectrum is

$$\mathcal{S}(\lambda) = \mathcal{S}_0(\lambda)\cos^2[\theta + \alpha(\lambda)], \tag{8}$$

where the total light intensity is given by the sum over wavelength, $\int \mathcal{S}_0(\lambda)\mathrm{d}\lambda = \mathcal{I}_0$.

The predicted transmission spectrum for a simulated input spectrum is illustrated in Fig. 2. The top row shows the input white-light spectrum. The second row shows the rotation predicted for a corn-syrup sample of depth 45 mm, and the bottom row displays the light transmitted with an analyzer set at $\theta = 0^\circ$. In this case, the zero intensity in the transmitted light occurs at the wavelength where the rotation, α, is 90°, which occurs at 525 nm. (Note that 120 mm of sucrose at a concentration at 90% of the saturation point would be required to give the same optical rotation at 525 nm.[16]) By changing the analyzer angle, θ, the location of the zero intensity can be moved through the spectrum, which is responsible for the color change effects observed in Fig. 1.

III. EXPERIMENTAL APPARATUS AND PROCEDURE

The equipment was set up on an optical rail in a dark box as shown in Fig. 3.

White light delivered by an optical fiber in an expanding light cone passed through an adjustable aperture and was collimated into a beam approximately 2 cm in diameter using a 50 mm lens. The collimated beam passed through a polarizer set to 0°, before striking the sample held in a square-sided glass bottle, approximately 45 mm each side, before passing though the analyzer (a second polarizer). Beyond this could be fitted either (i) the body of a Canon EOS 1300D SLR

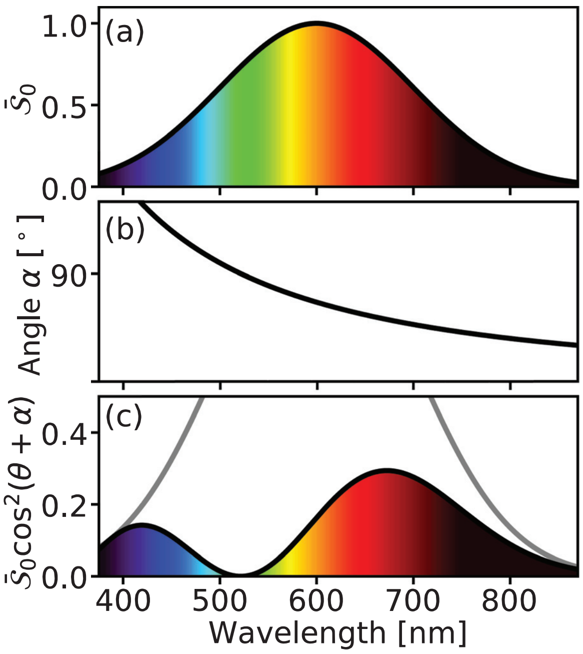

Fig. 2. (a) Simulated white-light source with normalized intensity I_0. (b) The optical rotation angle α for corn syrup of length 45 mm, using the rotation and dispersion constants from Ref. 13. (c) The measured light intensity (normalized) for an analyzer angle $\theta = 0°$.

camera with the lens removed, positioned such that the beam struck the CCD element directly, or (ii) the optical fiber of a spectrometer (IS Instruments miniature spectrometer) (Fig. 3). The experiment was conducted at room temperature.

Spectra and photographs of the transmitted light were recorded as the analyzer was rotated at 2° increments starting at 0° and finishing at 180°. The choice of sample length is important: if the cell is too long, there is too much absorption (especially at shorter wavelengths), and if it is too short, there is not enough rotation. As discussed above, our motivation for a sample length of 45 mm of corn syrup is that a rotation of 90° occurs at 525 nm, see Fig. 2(b).

For a given analyzer angle, a "calculated spectrum" was produced by applying Eqs. (5)–(7) to predict the expected rotation and transmission of each wavelength in the input white-light spectrum. For each of the calculated spectra, a corresponding color swatch was generated using the "colour" color science python library.[17] After binning the spectral data into 5 nm wide bins, a library function (`colour.spectral_to_XYZ`) was used to calculate the color in terms of XYZ (tristimulus) values. These values were readily converted into standard RGB color values from which the color swatches could be generated. Other library functions were used to extract the dominant RGB color information from each of the photographs corresponding to the measured spectra. Additionally, each of these color values was converted into "Lab" color space to facilitate color difference calculations using the `colour.delta_E` library function.

IV. RESULTS

Figure 4(a) shows typical spectra recorded without ($\mathcal{S}_0$, solid line) and with ($\mathcal{S}$, dashed line) corn syrup for an analyzer angle $\theta = 0°$. The "notch" in the transmission in the green—predicted in Section. II—is clearly visible. By plotting the ratio $\mathcal{S}/\mathcal{S}_0$, Fig. 4(b), we obtain Malus' law $\cos^2\alpha$. Optical rotary dispersion is evident as α is a function of λ. In principle, we could fit this curve to obtain the dispersive law. The fit is shown in Fig. 4(b); however, the fit parameters are too sensitive to systematics in the data to obtain the dispersion law with precise parameters in this way.

Instead, we focus on how the color of the transmitted light evolves as we rotate the analyzer. A gif file animation of the change in color as a function of the angle of the analyzer—θ

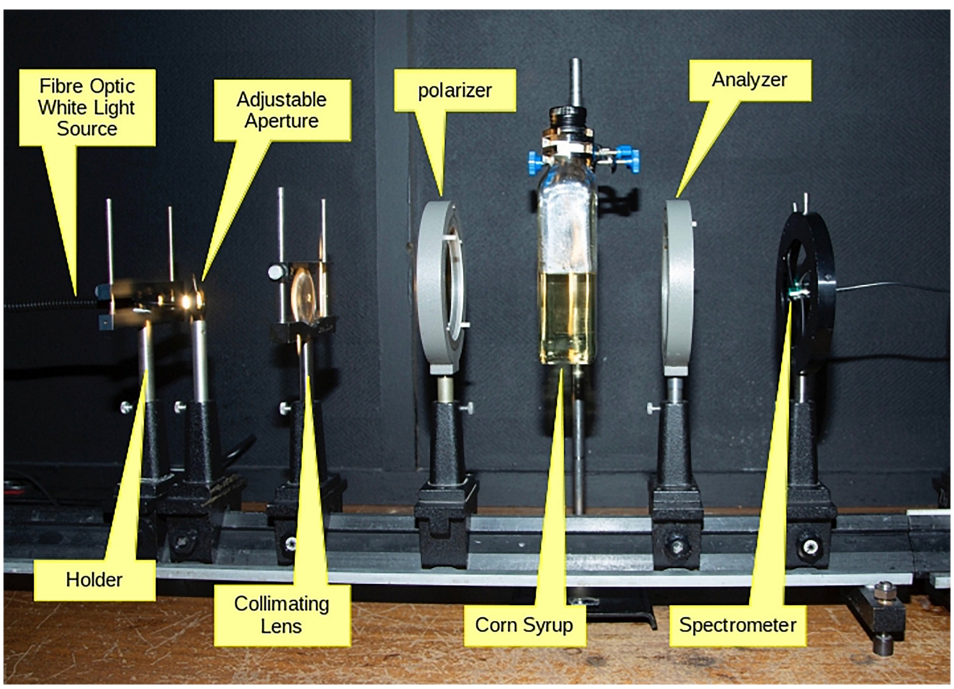

Fig. 3. A photograph of the optical setup. Light from a white-light source is collimated by a lens and then linearly polarized before passing through a glass bottle containing corn syrup. The transmitted light passes through a rotatable linear polarizer (the analyzer) and is then incident on either a camera or a multimode fiber connected to a spectrometer.

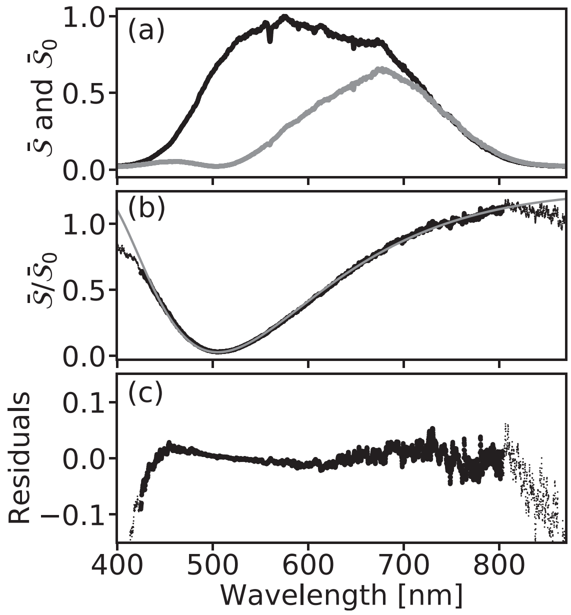

Fig. 4. (a) The spectrum, $\mathcal{S}_0$, recorded without any corn syrup in the bottle (black line) and the spectrum recorded with corn syrup (gray line) both for an analyzer angle $\theta = 0°$. (b) The ratio of with and without syrup as a function of wavelength with a Malus' law fit. The data used in the fit are highlighted using a larger marker size. (c) The residuals (difference between theory and experiment) for an analyzer angle $\theta = 0°$.

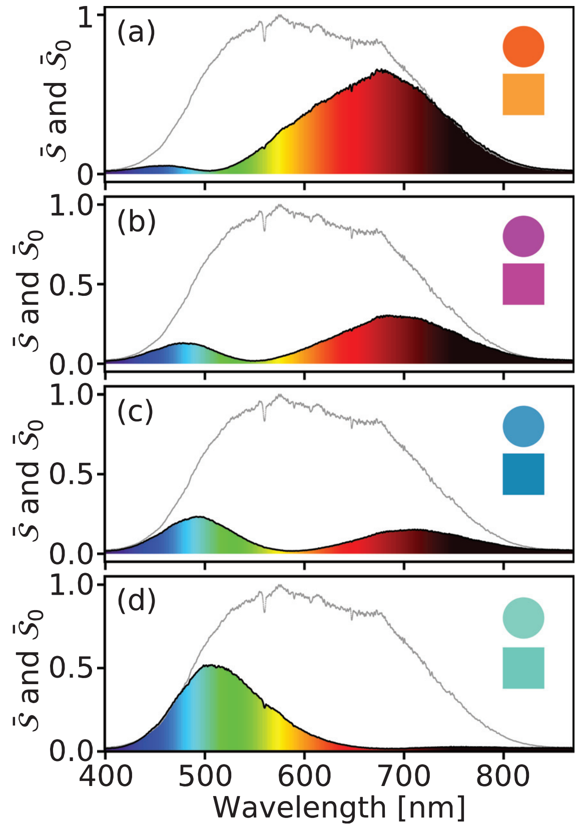

Fig. 5. The effect of the analyzer angle, θ, on the transmitted spectrum and the observed color. (a) $\theta = 0°$, (b) $\theta = 16°$, (c) $\theta = 26°$, and (d) $\theta = 44°$. The circle shows the color recorded using a camera. The square shows the color calculated using Malus' law. A gif file animation of the change in color as a function of the angle of the analyzer can be found in the supplementary material (Ref. 18).

in 2° increments—can be found in the supplementary material.[18] Figure 5 shows a selection of the data obtained using the spectrometer. The dramatic change of color of the transmitted beam as a function of analyzer angle is evident in the color image inset in each plot. The successive panels in Fig. 5 highlight the presence of a notch in the spectrum of the transmitted light. The location of maximum extinction evidently translates along the spectrum as the analyzer is rotated. Therefore, in contrast to filters that select colors based on absorption of light, the combination of two polarizers and a chiral medium allows the location of maximum extinction to be positioned at will within the spectrum.

Each panel in Fig. 5 has a marker to denote the transmitted color: the circle is an image obtained using the camera, while the square is the expected color calculated using the procedure outlined above. Using the values from Ref. 13 gives predicted spectra that are very similar to the measured ones; however, there is a slight discrepancy. Nixon and Hughes[13] assumed a corn syrup concentration of 1 g/ml, but the exact value for the sample used was not known.

We can use the RGB additive color model to explain the hues seen in Fig. 5. In panel (a), we see that extinguishing blue completely, green partially, and transmitting red leads to orange, as expected. Likewise, the magenta in (b) arises when blue and red are transmitted and green is suppressed. Suppressing red but transmitting blue and green, panel (d), generates cyan. Thus, we see that the somewhat mysterious sequence of colors generated by rotating the analyzer is explained by the location of the translatable region of maximum extinction within the spectrum.

A. Discussion

The differential rotation of the different colors present in white light produces a complicated change in color after traversing a chiral medium. However, the results presented here show that the simple Drude model of optical rotatory dispersion, along with Malus' law, fully accounts for the experimental observation. There is excellent agreement between theory and experiment for the spectral dependence of the rotation and the prediction of the observed color.

The analysis assumes that the plane of polarization of each color is rotated, but that there is no (differential) absorption. The excellent agreement between theory and experiment verifies the validity of this approximation for the experimental parameters used in this investigation. Nixon and Hughes[13] demonstrated that absorption is easily visible for longer sample lengths, see, in particular, Fig. 3 of Ref. 13 where the diminishing of intensity with propagation of blue light in a sample of length 6.5 dm is particularly prominent. Performing the experiment with a longer medium, and using a more sophisticated model taking into account circular dichroism, would potentially allow precise measurements of the parameters in the Drude expression by minimizing the difference between the experimental and predicted spectra.[19]

The missing notch in the transmitted spectrum reveals that the combination of two polarizers and a chiral medium can be used as a color filter. In this case, the light-matter interaction is spectrally broad, as a molecular resonance far from resonance is used. Using the same principle, but in the vicinity of an atomic resonance, enables the production of narrow optical bandpass filters[20–24] and optical isolators.[25]

Similar transmission "notches" can be generated in crystals displaying the Faraday effect and have been studied with white light to characterize the optical properties of crystals.[26]

V. CONCLUSION

Transmitting white light through a short length of corn syrup between crossed polarizers produces colored light. The spectrum is easily modified by rotating the angle of the analyzer. Excellent agreement is obtained between the experimentally measured spectra and those predicted for a simple model using the Drude equation for optical rotatory dispersion and Malus' law for transmission of polarized light by a polarizer. Whereas optical rotatory dispersion is conventionally studied in the undergraduate laboratory with a discrete number of fixed wavelengths,[5,13] this investigation demonstrates that a quantitative understanding of the same physics can be revealed in the optical rotation of white light.

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